EXTRA PROBLEM 2A

$$P = \frac{Y - Z}{X + Z}$$

$Y = 10.00 \pm 0.82$

$Z = 2.00 \pm 0.17$

$X = 12.00 \pm 0.55$

$$P = \frac{10.00 - 2.00}{12.00 + 2.00} = \underline{0.571}$$

SOLUTION #1

let $u = Y - Z$ $\qquad P = \frac{u}{v}$

$v = X + Z$

$u = 10.00 - 2.00 = 8.00$

$v = 12.00 + 2.00 = 14.00$

$$\sigma_u^2 = \sigma_y^2 + \sigma_z^2 = (0.82)^2 + (0.17)^2$$

$$\sigma_u = 0.837$$

$$\sigma_v^2 = \sigma_x^2 + \sigma_z^2 = (0.55)^2 + (0.17)^2$$

$$\sigma_v = 0.576$$

$$\left(\frac{\sigma_p}{P}\right)^2 = \left(\frac{\sigma_u}{u}\right)^2 + \left(\frac{\sigma_v}{v}\right)^2$$

$$\sigma_p^2 = P^2\left[\left(\frac{\sigma_u}{u}\right)^2 + \left(\frac{\sigma_v}{v}\right)^2\right]$$

$$= 0.576^2\left[\frac{0.837^2}{8.00^2} + \frac{0.576^2}{14.00^2}\right]$$

$$\sigma_p = 0.0643$$

$$P = \underline{0.57 \pm 0.06}$$

RELATIVE ERROR = 11.2%

SOLUTION #2

$$\frac{\partial \rho}{\partial y} = \frac{1}{x+z}$$

$$\frac{\partial \rho}{\partial x} = -\frac{y-z}{(x+z)^2}$$

$$\frac{\partial \rho}{\partial z} = -\frac{1}{x+z} - \frac{y-z}{(x+z)^2} = -\frac{x+y}{(x+z)^2}$$

$$\sigma_\rho^2 = \left(\frac{\partial \rho}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial \rho}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial \rho}{\partial z}\right)^2 \sigma_z^2$$

$$\sigma_\rho^2 = \left(\frac{y-z}{(x+z)^2}\right)^2 \sigma_x^2 + \left(\frac{1}{x+z}\right)^2 \sigma_y^2 + \left(\frac{x+y}{(x+z)^2}\right)^2 \sigma_z^2$$

$$\sigma_\rho^2 = \left(\frac{8}{196}\right)^2 (0.55)^2 + \left(\frac{1}{14}\right)^2 (0.82)^2 + \left(\frac{22}{196}\right)^2 (0.11)^2$$

$$\sigma_\rho^2 = 0.000504 + 0.003431 + 0.000364$$

$$\sigma_\rho = 0.0656$$

$$\rho = 0.57 \pm 0.07$$

RELATIVE ERROR = 11.5%